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A NEUTROSOPHIC MULTIVALUED DATA DEPENDENCY

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Abstract- This paper focus on Neutrosophic set theory in relational databases. It is known that functional dependency like multi valued dependency, play a key role in database design. We introduced a new definition of neutrospophic multivalued dependency (nmvd), called α-nmvd, on the basis of α-equality of tuples as defined in the recent work on neutrosophic functional dependency [1] in 2019. The definition has been shown and finally set of sound and complete inference axioms have been designed and verified for the α-nmvd.
Keywords: Neutrosophic set, Similarity measure of neutrosophic sets, α-equality of neutrosophic tuples, Neutrosophic functional dependency, Neutrosophic multivalued dependency

1. INTRODUCTION

The classical relational data model has been introduced by Codd [2] in 1970 can handle only precise and exact data of a source. But, information obtained from real life applications is more often imprecise. Vague set theory has been subsequently introduced by Gau and Buehrer [3] in1993, to deal with such uncertain information in relational databases.

Neutrosophic set has been introduced by Smarandache[4] in 2001 to deal with imprecise information in a more efficient manner than vague set theory in many different fields such as decision analysis, expert system, etc. using truth membership function tv, indeterminacy function iv and a false membership function fv.

Therefore, classical relational databases may be extended deal with uncertain data in a more effective manner with the neutrosophic set theory. Such an extended database model is called a neutrosophic relational database model. Research in this direction has attracted recently. However, till date very less numbers of references are available in this area. Integrity constraints are known to play a vital role in any database design as well as database manipulation. The study of different integrity constraints such as functional dependency, multivalued dependency, join dependency, inclusion dependency, etc., are generate a major area of research in database design. Thus, as one extends the classical relational database into a neutrosophic relational database, it is important to study these integrity constraints in vague set theory. We learnt a new vague functional dependency (called α-vfd) in reference [5, 6, 7]. But in the year 2019, De and Mishra [1] have developed more promising and effective Neutrosophic functional dependency (called α-nfd) particular based on the notion of α-equality of tuples and similarity measure of neutrosophic sets, it has been shown in reference [8, 9, 10,11, 12] that a neutrosophic relational database model may be more fruitful in processing uncertain data and queries than the conventional fuzzy and vague data models.

Multivalued dependency is another very important data integrity constraint to maintain consistency of a neutrosophic relational database. In the present work, we have made an attempt to extend the work on α-nfd and have proposed a new definition of neurosophic multivalued dependency. It is termed as α-VMVD and is defined on the basis of α-equality of tuples as introduced in [1]. Here we proposed a definition, actually provides a straight forward way of extending MVD to NMVD for a neutrosophic relational database model using the concept of α-equality of tuples. Inference axioms have also been proposed and proved for α-nmvd.

The paper is organized as follows: In Section 2, basic definitions of neutrosophic set theory have been revisited and similarity measure of neutrosophic sets have also been presented in the same section. In Section 3, the new definition of neutrosophic multivalued dependency (α-nmvd) has been proposed. The consistency as well as the inference axioms for our proposed α-nmvd have also been studied and proved in the same section. Finally, the concluding remarks appear in Section 4.

2. BASIC DEFINITIONS

In this section, we introduce the new concept of neutrosophic set. Let U be the universe of discourse where an element of U is denoted by u.

Definition 2.1

A neutrosophic set X on the universe of discourse U is characterized by three membership functions given by:

(i) a truth membership function $t_x : U \rightarrow [0,1]$,

(ii) a false membership function $f_x : U \rightarrow [0,1]$ and

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(iii) a indeterminacy membership function $i_x : U \to [0,1]$ such that $t_x(a) + f_x(a) \le 1$ and $t_x(a) + f_x(a) + i_x(a) \le 2$ and is written as $X = \{\langle x, [t_x(a), i_x(a), f_x(a)]\rangle, a \in U\}$.

Definition 2.2
A Neutrosophic set 'N' is an empty neutrosophic set, denoted by □ □, if and only if its truth-membership function tn □u□ □ □ 0□ indeterminacy-membership function in(u)= 0 and false-membership function fn □u□ □ □1 for all u on U.

Definition 2.3
A Neutrosophic set 'A' is contained in another neutrosophic set 'B', written as A □ □B, if and only if, tA □ □tB, iA □ □ □ □ □ □and fA □ □fB .

Definition 2.4
Two Neutrosophic sets 'A' and 'B' are equal, written as A = B, iff, A □ □B and B □ □A, that is, tA □ □tB , iA = iB and fA □ □fB .

Definition 2.5
The union of two neutrosophic sets A and B is a neutrosophic set C, denoted as C = A ∪□B, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A and B by tC = max(tA , tB), iC □ □min(iA , iB) and fC □ □min(fA , fB) .

Definition 2.6
The intersection of two Neutrosophic sets A and B is a neutrosophic set C, written as C = A∩□B, such that tC = min(tA , tB) , iC □ □max(iA , iB) and fC □ □max(fA , fB) .

Definition 2.7
The intersection of two neutrosophic sets A and B is a neutrosophic set C, written as C = A ∩ B, such that tC = min(tA, tB) , fC = max(fA, fB) and iC = max(iA, iB).

Definition 2.8
Let U = U1× U2 ×⋯× Un be the Cartesian product of n universes, and Ni, i = 1, 2, ⋯ , n be neutrosophic sets in their corresponding universe of discourses Ui, i = 1, 2, ⋯ , n respectively. Also, let ui ∈ Ui, i = 1, 2, ⋯ , n. Then the Cartesian product N = N1 × N2 ×⋯× Nn is defined to be a neutrosophic set of U = U1× U2 ×⋯× Un with the truth and false membership functions defined as follows:
tN (u1, u2 , ⋯ , un) = min(tN1 (u1), tN2 (u2), ⋯ , tNn (un)),
fN (u1, u2 , ⋯ , un) = max(tN1 (u1), tN2 (u2), ⋯ , tNn (un)),
iN (u1, u2 , ⋯ , un) = max(tN1 (u1), tN2 (u2), ⋯ , tNn (un)).

Definition 2.9
Let Ai, i = 1, 2, ⋯ , n be n attributes defined on the universes of discourse sets Ui, respectively. Then a neutrosophic relation r on the relation schema R(A1, A2, ⋯ , An) is defined as a subset of the Cartesian product of a collection of neutrosophic subsets: r ⊆ N(U1) × N(U2) ×⋯× N(Un), where N(Un) denotes the collection of all neutrosophic subsets on Ui.
Each tuple t of r consists of a Cartesian product of neutrosophic subsets on the respective Ui's, i.e., t[Ai] = π(Ai), where π(Ai) is a neutrosophic subset of the attribute Ai defined on Ui for all i. The relation r can thus be represented by a table with n columns. It may be observed that the neutrosophic relation can be considered as an extension of classical relations (all values are [1, 1]) and fuzzy relations (all neutrosophic values are [a, a, a], 0 ≤ a ≤ 1). It is clear that more information about fuzziness can be captured in a neutrosophic relation.

2.2. Similarity Measure of Neutrosophic Data

Let x and y be any two neutrosophic values such that $x = [t_x, i_x, f_x]$ and $y = [t_y, i_y, f_y]$ where $0 \le t_x \le 1, 0 \le i_x \le 1, 0 \le f_x \le 1$ and $0 \le t_y \le 1, 0 \le i_y \le 1, 0 \le f_y \le 1$ with $0 \le t_x + f_x \le 1$, $0 \le t_y + f_y \le 1$, $0 \le t_x + i_x + f_x \le 2$, $0 \le t_y + i_y + f_y \le 2$.

Now the similarity measure between two neutrosophic data denoted by $SE(x, y)$ is defined as follows

$$SE(x,y)=\sqrt{1-\frac{|(t_x-t_y)-(i_x-i_y)-(f_x-f_y)|}{3}\left(1-|(t_x-t_y)+(i_x-i_y)+(f_x-f_y)|\right)}$$

2.3. Neutrosophic Functional Dependency

The new definition of α-nfd proposed in Ref. [1] is based on the idea of α-equality of two neutrosophic tuples defined as follows:

Definition 2.10 (α-equality of two neutrosophic tuples)

Let r(R) be a vague relation on the relational schema R(A1, A2, ⋯ An). Let t1 and t2 be any two neutrosophic tuples in r. Let α ∈ [0, 1] be a threshold or choice parameter, predefined by the database designer, and X = {X1, X2, ⋯ Xk} ⊆ R. Then the neutrosophic tuples t1 and t2 are said to be α-equal on X if SE(t1[Xi], t2[Xi]) ≥ α, ∀i = 1, 2, 3, ⋯ k. This equality is denoted by the notation $t_1[X](NE)_\alpha t_2[X]$.

Then the new definition α-NFD [1] is given as follows:

Definition 2.11

Let X, Y ⊂ R = {A1, A2… An}. Choose a threshold value $\alpha \in [0,1]$. Then a neutrosophic functional dependency (α -nfd), denoted by $X \xrightarrow[\alpha]{nfd} Y$ is said to exist if, whenever $t_1[X](NE)_\alpha t_2[X]$, it is also the case that $t_1[Y](NE)_\alpha t_2[Y]$.

It may be read as “X neutrosophic functionally determines Y at α -level”. In another way, “Y is neutrosophic functionally determined by X at α –level”. The following proposition for α -nfd is straightforward.

The following propositions are straightforward from the above definitions.

Proposition 1

If $0 \le \alpha_2 \le \alpha_1 \le 1,$ then $t_1[X](NE)_{\alpha_1} t_2[X] \Rightarrow t_1[X](NE)_{\alpha_2} t_2[X]$

Proposition 2

If Y ⊆ X, then for any two tuples t1 and t2 in r and for any 0 ≤ α ≤ 1, $t_1[X](NE)_\alpha t_2[X] \Rightarrow t_1[Y](NE)_\alpha t_2[Y]$.

Proposition 3

If $0 \le \alpha_2 \le \alpha_1 \le 1,$ then $X \xrightarrow[\alpha_1]{nfd} Y \Rightarrow X \xrightarrow[\alpha_2]{nfd} Y$

3. MULTIVALUED DEPENDENCY

MVD, as introduced by Fagin [13], is an important data dependency, which helps the database designer to remove redundancy from the database. Informally, MVD in a database relates a value of an attribute (or a set of attributes) to a set of values associated with a set of attributes, independent of the other attributes in the relation.

Formally, the definition of MVD may be written as follows:

Definition 3.1

Let r be an instance of relational schema R (A1, A2, ⋯ , An) and let X, Y ⊂ R. Then X →→ Y (read as X multi determines Y) if for any two tuples t1 and t2 in r with t1[X] = t2[X], there exists a third tuple t3 in r with t1[X] = t2[X] = t3[X], t1[Y] = t3[Y] and t2[R − X − Y] = t3[R − X − Y].

3.1 Neutrosophic Multivalued Dependency

The problem associated with classical multivalued dependency for imprecise data may be resolved efficiently with the theory of neutrosophic sets. In this section, we present our new notion of neutrosophic multivalued dependency based on the idea of α-equality of tuples and is termed as α-NMVD. This extension to α-NMVD is straightforward where we use a similar procedure as adopted for α-NFD and soften the strict equality in MVD by α-equality as follows:

Definition 3.2

Let r be an instance of relational schema R(A1, A2, ⋯ , An) and let X, Y ⊂ R. Then, the relation r(R) satisfies X →→ Y at the level α (called X vague multi determines Y at the level α) is denoted by $X \rightarrow \xrightarrow[\alpha]{nmvd} Y$ and is defined as:

For any two tuples t1 and t2 in r, if $t_1[X](NE)_\alpha t_2[X]$, then there must exist a third tuple t3 in r with

$t_1[X](NE)_\alpha t_2[X](NE)_\alpha t_3[X]$,

$t_1[Y](NE)_\alpha t_3[Y]$,

$t_2\left[R-X-Y\right]\left(NE\right)_\alpha t_3\left[R-X-Y\right]$.

Next, we show the consistency of this new definition of α-NMVD. In order to do this, we prove that this definition of α-NMVD reduces to that of classical MVD at α = 1, i.e., the NMDV $X \rightarrow \xrightarrow[\alpha=1]{nmvd} Y \Rightarrow$ MVD X →→ Y .

Lemma 3.1 The definition of the α-NMVD is consistent.

Proof

Let us assume that $X \rightarrow \xrightarrow[\alpha=1]{nmvd} Y$. Then, from the definition of α-NMVD it follows that for any two tuples t1 and t2 in r, if $t_1\left[X\right]\left(NE\right)_{\alpha=1} t_2\left[X\right]$, then there must exist a third tuple t3 in r with

$$t_1\left[X\right]\left(NE\right)_{\alpha=1} t_2\left[X\right]\left(NE\right)_{\alpha=1} t_3\left[X\right], \quad (1)$$

$$t_1\left[Y\right]\left(NE\right)_{\alpha=1} t_3\left[Y\right], \quad (2)$$

$$t_2\left[R-X-Y\right]\left(NE\right)_{\alpha=1} t_3\left[R-X-Y\right]. \quad (3)$$

Now from Definition 2.10,

$t_1\left[X\right]\left(NE\right)_{\alpha=1} t_2\left[X\right]$

⇒ SE (t1[Xi], t2[Xi]) ≥ α, ∀i = 1, 2, ⋯ , k

⇒ SE (t1[Xi], t2[Xi]) ≥ 1, ∀i = 1, 2, ⋯ , k (since α = 1)

⇒ SE (t1[Xi], t2[Xi]) = 1, ∀i = 1, 2, ⋯ , k (since SE(t1[Xi], t2[Xi]) ∈ [0, 1])

⇒ t1[Xi] = t2[Xi], ∀i = 1, 2, ⋯ , k

⇒ t1[X] = t2[X].

Using this in the above relations (1), (2) and (3), we see that for any two tuples t1 and t2 in r, if t1[X] = t2[X], then there must exist a tuple t3 in r with

t1[X] = t2[X] = t3[X],

t1[Y] = t3[Y],

t2[R − X − Y] = t3[R − X − Y],

which implies X →→ Y.

Thus, our definition of α-NMVD with α = 1 is equivalent to the classical definition of MVD, and hence the definition is consistent.

3.2 Inference Rules for α-NMVD

In classical relational database, multivalued dependency is known to satisfy a set of inference rules. In this section, we propose and prove the following inference rules for the α-NMVD.

(R3.1) Replication rule for α-NMVD (α-NMVD-replication):

If $X \xrightarrow[\alpha]{nfd} Y$, then $X \rightarrow \xrightarrow[\alpha]{nmvd} Y$.

(R3.2) Complementation rule for α-NMVD (α-NMVD-complementation):

If $X \rightarrow \xrightarrow[\alpha]{nmvd} Y$, then $X \rightarrow \xrightarrow[\alpha]{nmvd} R-X-Y$.

(R3.3) Inclusion rule for α-NMVD (α-NMVD-inclusion):

If $X \rightarrow \xrightarrow[\alpha_1]{nmvd} Y$ and $\alpha_1 \ge \alpha_2$, then $X \rightarrow \xrightarrow[\alpha_2]{nmvd} Y$.

(R3.4) Union rule for α-NMVD (α-NMVD-union):

If $X \rightarrow \xrightarrow[\alpha_1]{nmvd} Y$ and $X \rightarrow \xrightarrow[\alpha_2]{nmvd} Z$, then $X \rightarrow \xrightarrow[\min(\alpha_1,\alpha_2)]{nmvd} YZ$.

(R3.5) Reflexivity rule for α-NMVD (α-NMVD-reflexivity):

If Y ⊆ X, then $X \rightarrow \xrightarrow[\alpha]{nmvd} Y$.

(R3.6) Projectivity rule for α-NMVD (α-NMVD-projectivity):

If $X \rightarrow \xrightarrow[\alpha_1]{nmvd} Y$ and $X \rightarrow \xrightarrow[\alpha_2]{nmvd} Z$, then $X \rightarrow \xrightarrow[\min(\alpha_1,\alpha_2)]{nmvd} Y-Z$, $X \rightarrow \xrightarrow[\min(\alpha_1,\alpha_2)]{nmvd} Z-Y$ and $X \rightarrow \xrightarrow[\min(\alpha_1,\alpha_2)]{nmvd} Y \cap Z$.

(R3.7) Augmentation rule for α-NMVD (α-NMVD-augmentation):

If $X \rightarrow \xrightarrow[\alpha]{nmvd} Y$ and W ⊆ U , then $UX \rightarrow \xrightarrow[\alpha]{nmvd} YW$

(R3.8) Transitivity rule for α-NMVD (α-NMVD-transitivity):

If $X \xrightarrow[\alpha_1]{nmvd} Y$ and $Y \xrightarrow[\alpha_2]{nmvd} Z$, then $X \xrightarrow[\min(\alpha_1,\alpha_2)]{nmvd} Z - Y$.

In the following sub-sections, we present a formal proof for each of the axioms proposed for our α-NMVD.

(R3.1) α-NMVD-replication rule:

If $X \xrightarrow[\alpha]{nfd} Y$, then $X \xrightarrow[\alpha]{nmvd} Y$.

Proof

We prove the replication rule by contradiction. Suppose some relation instance r of R satisfies $X \xrightarrow[\alpha]{nfd} Y$ but violates $X \xrightarrow[\alpha]{nmvd} Y$. Then, for tuples t1 and t2 in r with $t_1[X](NE)_\alpha t_2[X]$, there exists a third tuple t3 in r for which $t_1[X](NE)_\alpha t_2[X](NE)_\alpha t_3[X]$ hold, but $t_1[Y](NE)_\alpha t_3[Y]$, and $t_2[R-X-Y](NE)_\alpha t_3[R-X-Y]$ do not hold.

Now since $X \xrightarrow[\alpha]{nfd} Y$ holds, then by Definition 2.11, we have for any tuples t1 and t3 in r, whenever $t_1[X](NE)_\alpha t_3[X]$, it also implies $t_1[Y](NE)_\alpha t_3[Y]$ which contradicts our assumption. Hence $X \xrightarrow[\alpha]{nmvd} Y$ Y holds.

(R3.2) α-NMVD-complementation rule:

If $X \xrightarrow[\alpha]{nmvd} Y$, then $X \xrightarrow[\alpha]{nmvd} R - X - Y$.

Proof

Given $X \xrightarrow[\alpha]{nmvd} Y$. Then, from the definition of α-NMVD, it follows that for any two tuples t1 and t2 with $t_1[X](NE)_\alpha t_2[X]$, there exists a tuple t3 such that

$$t_1[X](NE)_\alpha t_2[X](NE)_\alpha t_3[X], \qquad (4)$$

$$t_1[Y](NE)_\alpha t_3[Y], \qquad (5)$$

$$t_2[R-X-Y](NE)_\alpha t_3[R-X-Y]. \qquad (6)$$

Since R − (R − X − Y) − X ⊆ Y, so from (5) by using Proposition 2.2, we can say that

$$t_2[R-(R-X-Y)-X](NE)_\alpha t_3[R-(R-X-Y)-X]. \qquad (7)$$

Then the α-NMVD-complementation rule follows from the relations (4), (6) and (7).

(R3.3) α-NMVD-inclusion rule:

If $X \xrightarrow[\alpha_1]{nmvd} Y$ and $\alpha_1 \geq \alpha_2$, then $X \xrightarrow[\alpha_2]{nmvd} Y$.

Proof

This rule follows directly from Proposition 2.1.

(R3.4) α-NMVD-union rule:

If $X \xrightarrow[\alpha_1]{nmvd} Y$ and $X \xrightarrow[\alpha_2]{nmvd} Z$, then $X \xrightarrow[\min(\alpha_1,\alpha_2)]{nmvd} YZ$.

Proof α1 ≥ α2.

Given $X \xrightarrow[\alpha_1]{nmvd} Y$ and $X \xrightarrow[\alpha_2]{nmvd} Z$.

Now $X \xrightarrow[\alpha_1]{nmvd} Y \Rightarrow X \xrightarrow[\alpha_2]{nmvd} Y$ by α-NMVD- inclusion rule.

Further, from the definition of $X \xrightarrow[\alpha_2]{nmvd} Y$, we have

$$t_1[X](NE)_\alpha t_2[X](NE)_\alpha t_3[X], \qquad (8)$$

$$t_1[Y](NE)_\alpha t_3[Y], \qquad (9)$$

$$t_2[R-X-Y](NE)_\alpha t_3[R-X-Y]. \qquad (10)$$

and from $X \xrightarrow[\alpha_2]{nmvd} Z$, we have

$$t_1[X](NE)_\alpha t_3[X](NE)_\alpha t_4[X], \qquad (11)$$

$t_1[Y](NE)_\alpha t_4[Y]$, (12)

$t_3[R-X-Y](NE)_\alpha t_4[R-X-Y]$. (13)

Then, from the relations (8) and (11), we have

$t_1[X](NE)_\alpha t_2[X](NE)_\alpha t_3[X](NE)_\alpha t_4[X]$. (14)

Next, if X, Y and Z are disjoint sets, then clearly Y ⊆ R − X − Z, so that from (13) by using Proposition 2.2, we may write

$t_3[Y](NE)_{\alpha_2} t_4[Y]$. (15)

Now, combining the relations (9) and (15), we get

$t_1[Y](NE)_{\alpha_2} t_4[Y]$. (16)

However, if the sets X, Y and Z are non-disjoint sets, let us consider the set Y, such that Y = Y − (Y ∩ X) − (Y ∩ Z).

Next, from the relations (11) and (12), we have $t_1[X](NE)_{\alpha_2} t_4[X]$ and $t_1[Z](NE)_{\alpha_2} t_4[Z]$, so that combining the above two relations, we may write

$t_1[XZ](NE)_{\alpha_2} t_4[XZ]$. (17)

Then from Equations (12) and (16), we have

$t_1[YZ](NE)_{\alpha_2} t_4[YZ]$. (18)

Again, since R − X − Y − Z ⊆ R − X − Y and R − X − Y − Z ⊆ R − X − Z, so from (10) and (13) by using Proposition 2.2, we get respectively

$t_2[R-X-Y-Z](NE)_{\alpha_2} t_3[R-X-Y-Z]$ and

$t_3[R-X-Y-Z](NE)_{\alpha_2} t_4[R-X-Y-Z]$.

From these two relations, we may write

$t_2[R-X-Y-Z](NE)_{\alpha_2} t_4[R-X-Y-Z]$. (19)

Hence, using the relations (14), (18) and (19), we have for any two tuples t1 and t2, if $t_1[X](NE)_{\alpha_2} t_2[X]$, then there exists a tuple t3 for which

$t_1[X](NE)_\alpha t_2[X](NE)_\alpha t_4[X]$,

$t_1[YZ](NE)_{\alpha_2} t_4[YZ]$,

$t_2[R-X-Y-Z](NE)_{\alpha_2} t_4[R-X-Y-Z]$,

$X \rightarrow \xrightarrow[\alpha_2]{nmvd} YZ$ [since R − X − Y − Z = R − X − YZ].

(R3.5) α-NMVD-reflexivity rule:

If Y ⊆ X, then $X \rightarrow \xrightarrow[\alpha]{nmvd} Y$.

Proof

Given Y ⊆ X. Then from α-NFD reflexive rule of Ref. [1], it follows that $X \xrightarrow[\alpha]{nfd} Y$. Again, from α-NMVD-replication rule, if $X \xrightarrow[\alpha]{nfd} Y$, then $X \rightarrow \xrightarrow[\alpha]{nmvd} Y$.

Hence, the α-NMVD-reflexivity rule is proved.

(R3.6) α-NMVD-projectivity rule:

If $X \rightarrow \xrightarrow[\alpha_1]{nmvd} Y$ and $X \rightarrow \xrightarrow[\alpha_2]{nmvd} Z$, then $X \rightarrow \xrightarrow[\min(\alpha_1,\alpha_2)]{nmvd} Y-Z$, $X \rightarrow \xrightarrow[\min(\alpha_1,\alpha_2)]{nmvd} Z-Y$ and $X \rightarrow \xrightarrow[\min(\alpha_1,\alpha_2)]{nmvd} Y \cap Z$.

Proof α1 ≥ α2.

We have $X \xrightarrow[\alpha_1]{nmvd} Y$ which implies $X \xrightarrow[\alpha_2]{nmvd} Y$ (By α-NMVD-inclusion rule).

Now, from the definition of $X \xrightarrow[\alpha_2]{nmvd} Y$, we have

$$t_1[X](NE)_{\alpha_2} t_2[X](NE)_{\alpha_2} t_3[X], \tag{20}$$

$$t_1[Y](NE)_{\alpha_2} t_3[Y], \tag{21}$$

$$t_1[R-X-Y](NE)_{\alpha_2} t_3[R-X-Y]. \tag{22}$$

Also from $X \xrightarrow[\alpha_2]{nmvd} Z$, we have

$$t_1[X](NE)_{\alpha_2} t_3[X](NE)_{\alpha_2} t_4[X], \tag{23}$$

$$t_1[Z](NE)_{\alpha_2} t_4[Z], \tag{24}$$

$$t_3[R-X-Y](NE)_{\alpha_2} t_4[R-X-Y]. \tag{25}$$

Again, from α-NMVD-reflexivity rule, we have $X \xrightarrow[\alpha_2]{nmvd} X$ (since X ⊆ X), which implies

$$t_1[X](NE)_{\alpha_2} t_2[X](NE)_{\alpha_2} t_3[X],$$

$$t_2[R-X](NE)_{\alpha_2} t_3[R-X], \tag{26}$$

or

$$t_1[X](NE)_{\alpha_2} t_3[X](NE)_{\alpha_2} t_4[X], \tag{27}$$

$$t_3[R-X](NE)_{\alpha_2} t_4[R-X]. \tag{28}$$

Now, from (1), we have

$$t_1[X](NE)_{\alpha_2} t_2[X](NE)_{\alpha_2} t_3[X]. \tag{29}$$

Since Y − Z ⊆ Y, so from (21) by using Proposition 2, we get

$$t_1[Y-Z](NE)_{\alpha_2} t_3[Y-Z]. \tag{30}$$

Also, since R − X − (Y − Z) ⊆ R − X, so from (26) by using Proposition 2, we get

$$t_2[R-X-(Y-Z)](NE)_{\alpha_2} t_3[R-X-(Y-Z)]. \tag{31}$$

Using relations (29), (30) and (31), we get $X \xrightarrow[\alpha_2]{nmvd} Y-Z$ (proved).

Again from (4), we have

$$t_1[X](NE)_{\alpha_2} t_3[X](NE)_{\alpha_2} t_4[X]. \tag{32}$$

Since Z − Y ⊆ Z, so from (24) by using Proposition 2.2, we get

$$t_1[Z-Y](NE)_{\alpha_2} t_4[Z-Y]. \tag{33}$$

Also, R − X − (Z − Y) ⊆ R − X, so from (28) by using Proposition 2.2, we get

$$t_3[R-X-(Z-Y)](NE)_{\alpha_2} t_4[R-X-(Z-Y)]. \tag{34}$$

Using relations (32), (33) and (34), we get $X \xrightarrow[\alpha_2]{nmvd} Z-Y$ (proved).

Next, from relation (32), we have

$$t_1[X](NE)_{\alpha_2} t_3[X](NE)_{\alpha_2} t_4[X]. \tag{35}$$

Since Z ∩ Y ⊆ Z, so from (28) by using Proposition 2.2, we get

$$t_1[Z \cap Y](NE)_{\alpha_2} t_4[Z \cap Y]. \tag{36}$$

Also, R − X − (Z ∩ Y) ⊆ R − X, so from (33) by using Proposition 2.2, we get

$$t_3\left[R-X-\left(Z\cap Y\right)\right]\left(NE\right)_{\alpha_2} t_4\left[R-X-\left(Z\cap Y\right)\right]. \quad (37)$$

Using relations (35), (36) and (37), we get $X \xrightarrow[\alpha_2]{nmvd} Z\cap Y$ (proved).

Hence, for given, $X \xrightarrow[\alpha_1]{nmvd} Y$ and $X \xrightarrow[\alpha_2]{nmvd} Z$, we have proved $X \xrightarrow[\alpha_2]{nmvd} Y-Z$, $X \xrightarrow[\alpha_2]{nmvd} Z-Y$ and $X \xrightarrow[\alpha_2]{nmvd} Y\cap Z$ where $\alpha 1 \geq \alpha 2$.

(R3.7) α-NMVD-augmentation rule:

If $X \xrightarrow[\alpha]{nmvd} Y$ and W ⊆ U , then $UX \xrightarrow[\alpha]{nmvd} YW$.

Proof Given $X \xrightarrow[\alpha]{nmvd} Y$, so that from the definition of α-NMVD we can say that for any two tuples t1 and t2, if $t_1\left[X\right]\left(NE\right)_{\alpha} t_2\left[X\right]$, then there exists a tuple t3 such that

$$t_1\left[X\right]\left(NE\right)_{\alpha} t_2\left[X\right]\left(NE\right)_{\alpha} t_3\left[X\right], \quad (38)$$

$$t_1\left[Y\right]\left(NE\right)_{\alpha} t_3\left[Y\right], \quad (39)$$

$$t_2\left[R-X-Y\right]\left(NE\right)_{\alpha} t_3\left[R-X-Y\right]. \quad (40)$$

Again, since W ⊆ U so by α-NMVD-reflexivity rule, we have $U \xrightarrow[\alpha]{nmvd} W$.

Hence, from the definition of α-NMVD, we have

$$t_1\left[U\right]\left(NE\right)_{\alpha} t_2\left[U\right]\left(NE\right)_{\alpha} t_3\left[U\right], \quad (41)$$

$$t_1\left[W\right]\left(NE\right)_{\alpha} t_3\left[W\right], \quad (42)$$

$$t_2\left[R-U-W\right]\left(NE\right)_{\alpha} t_3\left[R-U-W\right]. \quad (43)$$

Now combining relations (38) and (41), we have

$$t_1\left[UX\right]\left(NE\right)_{\alpha} t_2\left[UX\right]\left(NE\right)_{\alpha} t_3\left[UX\right]. \quad (44)$$

Also combining relations (38) and (42), we have

$$t_1\left[YW\right]\left(NE\right)_{\alpha} t_3\left[YW\right]. \quad (45)$$

Again since R − UX − YW ⊆ R − X − Y or R − UX − YW ⊆ R − U − W, so either from (40) or (43) by using Proposition 2.2, we have

$$t_2\left[R-UX-YW\right]\left(NE\right)_{\alpha} t_3\left[R-UX-YW\right]. \quad (46)$$

Therefore, relations (44), (45) and (46) imply $UX \xrightarrow[\alpha]{nmvd} YW$ and the augmentation rule is proved.

(R3.8) α-NMVD-transitivity rule:

If $X \xrightarrow[\alpha_1]{nmvd} Y$ and $Y \xrightarrow[\alpha_2]{nmvd} Z$, then $X \xrightarrow[\min(\alpha_1,\alpha_2)]{nmvd} Z-Y$.

Proof $\alpha 1 \geq \alpha 2$

Given $X \xrightarrow[\alpha_1]{nmvd} Y$ and $Y \xrightarrow[\alpha_2]{nmvd} Z$.

Now $X \xrightarrow[\alpha_1]{nmvd} Y \Rightarrow X \xrightarrow[\alpha_2]{nmvd} Y$ by α-NMVD- inclusion rule.

Then, from the definition of $X \xrightarrow[\alpha_2]{nmvd} Y$, we have

$$t_1\left[X\right]\left(NE\right)_{\alpha_2} t_2\left[X\right]\left(NE\right)_{\alpha_2} t_3\left[X\right], \quad (47)$$

$$t_1\left[Y\right]\left(NE\right)_{\alpha_2} t_3\left[Y\right], \quad (48)$$

$$t_2\left[R-X-Y\right]\left(NE\right)_{\alpha_2} t_3\left[R-X-Y\right]. \quad (49)$$

and from $Y \xrightarrow[\alpha_2]{nmvd} Z$, we have

$$t_1[Y](NE)_{\alpha_2}\, t_2[Y](NE)_{\alpha_2}\, t_3[Y], \tag{50}$$

$$t_1[Z](NE)_{\alpha_2}\, t_3[Z], \tag{51}$$

$$t_2[R-Y-Z](NE)_{\alpha_2}\, t_3[R-Y-Z]. \tag{52}$$

Now combining relations (50) and (51), we have

$$t_1[YZ](NE)_{\alpha_2}\, t_4[YZ]. \tag{53}$$

Next, if X, Y and Z are disjoint sets, then X ⊆ R − Y − Z, so from (52) by using Proposition 2, we get

$$t_3[X](NE)_{\alpha_2}\, t_4[X]. \tag{54}$$

However, if the sets X, Y and Z are non-disjoint, such that
X = X − (X ∩ Z) − (X ∩ Y).
Then, X ⊆ R − Y − Z, so from (52) by using Proposition 2.2, we get

$$t_3[X](NE)_{\alpha_2}\, t_4[X]. \tag{55}$$

Again, since X − X ⊆ X, so from (42) by using Proposition 2.2, we get

$$t_1[X-X](NE)_{\alpha_2}\, t_2[X-X](NE)_{\alpha_2}\, t_3[X-X]. \tag{56}$$

Further, X − X ⊆ YZ, so from (53) by using Proposition 2.2, we get

$$t_1[X-X](NE)_{\alpha_2}\, t_4[X-X]. \tag{57}$$

Then from (47) and (54), we have

$$t_1[X](NE)_{\alpha_2}\, t_2[X](NE)_{\alpha_2}\, t_3[X](NE)_{\alpha_2}\, t_4[X]. \tag{58}$$

Further, since R − X − Y − Z ⊆ R − X − Y as well as R − X − Y − Z ⊆ R − Y − Z, hence from (49) and (52) using Proposition 2.2, we get respectively

$$t_2[R-X-Y-Z](NE)_{\alpha_2}\, t_3[R-X-Y-Z]. \tag{59}$$

and

$$t_3[R-X-Y-Z](NE)_{\alpha_2}\, t_4[R-X-Y-Z]. \tag{60}$$

Combining the two relations (59) and (60) above, we may write

$$t_2[R-X-Y-Z](NE)_{\alpha_2}\, t_4[R-X-Y-Z]. \tag{61}$$

Thus, using the relations (53), (58) and (61), we may say, for any two tuples t1 and t2,

if $t_1[X](NE)_{\alpha_2}\, t_2[X]$, then there exists a tuple t4 for which

$$t_1[X](NE)_{\alpha_2}\, t_2[X](NE)_{\alpha_2}\, t_4[X],$$

$$t_1[YZ](NE)_{\alpha_2}\, t_4[YZ],$$

$$t_2[R-X-Y-Z](NE)_{\alpha_2}\, t_4[R-X-Y-Z],$$

which implies $X \to \xrightarrow[\alpha_2]{nmvd} YZ$.

Hence, $X \to \xrightarrow[\alpha_1]{nmvd} Y$ and $Y \to \xrightarrow[\alpha_2]{nmvd} Z$, then $X \to \xrightarrow[\alpha_2]{nmvd} YZ$, where α1 ≥ α2.

4. CONCLUSION

MVD for functional dependency and constitutes an integrity about data constraint. In the present paper, we have extended the work on neutrosophic functional dependency in refer [1] to present a new notion of α-NMVD with the idea of α-equality of tuples. Here, present definition provides a straightforward way of extension of MVD to NMVD and is the first of its kind in literature in the context of a neutrosophic relational database. A set of inference axioms have also been investigated for the α-NMVD which play an important role in checking integrity constraints.

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